5.1 – Exponential Equations

Lesson Objectives

1. Students will be able to write explicit equations for geometric sequences.
2. Students will be able to define exponential functions, recognize $y = ab^x$ as the parent function, and be able to distinguish them from linear functions and models.
3. Students will see real-world growth and decay situations and model them with an exponential equation.
4. Students will recognize that the exponential function models growth when b is greater than 1 and models decay when b is less than 1.
5. Students will be able to evaluate an exponential function using both explicit equations and graphical models and be able to gauge the reasonableness of their answer and interpret the solution.

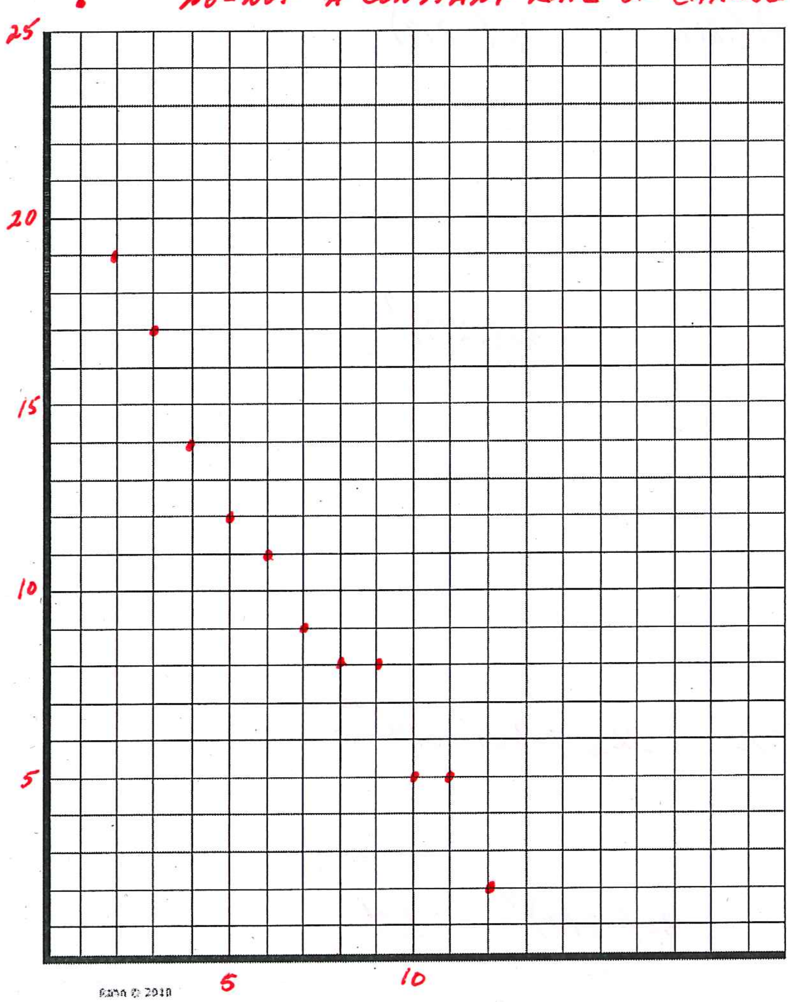
Investigation: Radioactive Decay

Procedure:

1. All members of the class should stand up
2. Each standing person rolls a die. Anyone who gets a 1 sits down.
3. Record the number of people standing.
4. Repeat steps 2 and 3 until fewer than 3 students are standing.

Step 1: Create a table of the information gathered in the investigation.
Step 2: Graph your data. The graph should remind you of the sequence graphs from Chapter 1. Does the data appear to be linear? How can you tell? What type of sequence does it resemble?

NO - NOT A CONSTANT RATE OF CHANGE   GEOMETRIC (DECREASING)



Times Die is Rolled	People Standing	Common Ratio
0	30	–
1	26	.86
2	19	.731
3	17	.895
4	14	.824
5	12	.857
6	11	.917
7	9	.818
8	8	.889
9	8	1
10	5	.625
11	5	1
12	2	.4
13		
14		
15		
16		

Step 2: Determine the ratio between each consecutive count and then average those values to determine your common ratio (r).

r = .819

Step 3: Identify u_0 and the common ration r, for your sequence. Complete the table like the table below:

n	u_n	u_n in terms of u_0 and r	u_n in terms of u_0 and r using exponents
0	u_0		
1	u_1	$u_0 * r$	$u_0 * r^1$
2	u_2	$u_0 * r * r$	$u_0 * r^2$
3	u_3	$u_0 * r * r * r$	$u_0 * r^3$

n	u_n	u_n in terms of u_0 and r	u_n in terms of u_0 and r using exponents
0	u_0		
1	u_1	$30(.819)$	$30(.819)^1$
2	u_2	$30(.819)(.819)$	$30(.819)^2$
3	u_3	$30(.819)(.819)(.819)$	$30(.819)^3$
4	u_4		
5	u_5		
6	u_6		
7	u_7		$30(.819)^7$
8	u_8		
9	u_9		
10	u_{10}		

4. What is or explicit equation (exponential model) for our situation?

$u_n = u_0 r^n$ $\qquad$ $u_n = 30(.819)^n$

5. Based on our model, how many students should be standing after rolling the die seven times?

$30(.819)^7 = 7.7$

6. Based on the result from #5, does our model is a good representation for the situation? Why or why not?

7 = 9 in graph

YES, POINTS PLOTTED WON'T FOLLOW GRAPH PERFECTLY BUT SHOULD BE CLOSE

Exponential Function

The general form, or intercept form, of an exponential function is

$y = ab^x$

where the coefficient a is the y-intercept and the base b is the growth rate.

Besides being the y-intercept, what will a represent in any exponential function?

The starting value

Example 1: What is the growth rate (or multiplier) if the function is growing by 4.5%? Decaying by 37.5%?

$1 + .045 = 1.045$ $\quad 1 - .375 = .625$

Example 2: Most automobiles depreciate as they get older. Suppose an automobile that originally costs $14,000 depreciates by 11% of its value every year.

a. What are the two variables in that will be used in the exponential model?

y = current cost (at x years)
x = # of years

b. Write the equation for the exponential function of this situation.

$y = 14{,}000(.89)^x$

c. What is the value of this automobile after 5 years?

$y = 14{,}000(.89)^5$
$7817.68

d. Convert this algebraic model to a graphical model using your calculator and determine when the car will lose half its value.

$7000 = 14{,}000(.89)^x$
$x = 5.948$ years

Example 3: A rare coin in Jo's coin collection is now worth $450. The value has been increasing by 15% each year.

a. Write the equation for the exponential function that models this situation.

$y = 450(1 + .15)^x$ $\quad y = 450(1.15)^x$

b. What will the value of the coin be in 11 ½ years?

$y = 450(1.15)^{11.5}$
$y = \$2245.11$